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IIB/M Duality and Longitudinal Membranes in M(atrix) Theory

Pei-Ming Ho and Yong-Shi Wu

*Department of Physics, University of Utah
Salt Lake City, Utah 84112*

Abstract

In this paper we study duality properties of the M(atrix) theory compactified on a circle. We establish the equivalence of this theory to the strong coupling limit of type IIB string theory compactified on a circle. In the M(atrix) theory context, our major evidence for this duality consists of identifying the BPS states of IIB strings in the spectrum and finding the remnant symmetry of $SL(2, Z)$ and the associated τ moduli. By this IIB /M duality, a number of insights are gained into the physics of longitudinal membranes in the infinite momentum frame. We also point out an accidental affine Lie symmetry in the theory.

1 Introduction

The recent revelation of various string dualities indicates clearly that all the previously known five consistent perturbative superstring theories in ten dimensions are closely related to each other; they appear to represent five corners of the moduli space for vacua of the one and same theory, which yet in another corner of the moduli space is most conveniently viewed as a theory in eleven dimensions, dubbed M theory. (For recent reviews see, e.g., [1]–[6].) Banks, Fischler, Shenker and Susskind [7] have proposed a definition of M theory in the infinite momentum frame (IMF) as a large N limit of maximally supersymmetric quantum mechanics of $N \times N$ matrices describing D0-branes. This proposal has passed a number of consistency tests [7],[8]–[21]. Many of the tests consist of verifying that M(atr)ix theory indeed reproduces the right properties and interactions of D p -branes (with even p) expected from type IIA string theory. And others examine the way certain string dualities are realized upon compactification on tori or more complicated spaces, and/or address some other issues.

The IMF description is essential to this formulation of M(atr)ix theory, which lacks manifest Lorentz invariance in eleven dimensions, and may give rise to some technical complications. For example, T-duality is not manifest, and nobody has succeeded in constructing a membrane wrapped in the longitudinal direction that defines the IMF. Our paper is devoted to a study of the M(atr)ix theory compactified on a circle, and of its duality properties closely related to type IIB strings, which was motivated by an intention for better understanding physics in the IMF.

The basic idea is the following. The uncompactified M(atr)ix theory is supposed to be equivalent to the strong coupling limit of type IIA string theory. Therefore, combining with the well-known IIA /IIB duality, the M(atr)ix theory *compactified on a circle* should be equivalent to the *strong coupling limit* of type IIB superstring theory compactified on a circle. Note that the IIB /M duality we are talking about here is *not* the usual one [22, 1], which involves compactifying M theory on a transverse torus. The IIB /M duality examined in this paper will allow us to infer the properties of longitudinal membranes.

For this duality in the context of M(atr)ix theory, we will provide the following evidence. First, we will identify, in the IMF spectrum of M(atr)ix theory, excitations corresponding to the BPS states of IIB strings. They include oscillation modes on the D-string resulting from compactification which, according to IIB /M duality, can be identified as excitations of a longitudinal membrane. Then, we will show that relations among the parameters of equivalent IIA, IIB and M theories (all compactified on a

circle) dictated by duality of each pair are satisfied in M(atrrix) theory. Moreover, we will show that the remnant symmetry of the celebrated S-duality $SL(2, Z)$ for IIB strings in the IMF is a subgroup isomorphic to the group of all integers; and we identify the τ moduli of the strongly coupled IIB string theory with the geometric tilt of the circle on which M(atrrix) theory is compactified.

Finally we present an observation that when compactified on a circle, the M(atrrix) theory action possesses not only a $U(N)$ loop group symmetry, but also the full affine Lie group symmetry, which is the central extension of the loop group, even at the *classical* level.

From this study, a number of insights are gained into the IMF description of the properties of longitudinal membranes in M(atrrix) theory. Many of them can be inferred from the properties of the D-strings. (At the end of the paper, an appendix is devoted to the description of moduli-dependent aspects of compactification on a slanted transverse torus.)

2 IIB /M Duality Revisited

Let us first consider what we would expect to happen in M theory, in accordance with IIB /M duality [23, 22], when compactified on a transverse circle in the IMF.

The spectrum of the type IIB theory compactified on a circle of radius R_B for a string of NS-NS and RR charges (q_1, q_2) , with q_1, q_2 mutually prime, is

$$M_B^2 = \left(\frac{n}{R_B}\right)^2 + \left(2\pi R_B T_{(q_1, q_2)} l\right)^2 + 4\pi T_{(q_1, q_2)} (N_R + N_L). \quad (1)$$

The first term is the contribution from the Kaluza-Klein excitations, the second term from the winding modes and the third term from the string oscillation modes. The tension of the (q_1, q_2) string is

$$T_{(q_1, q_2)} = \left((q_1 - \chi_0 q_2)^2 + g_B^{-2} q_2^2\right)^{1/2} T_s, \quad (2)$$

where χ is the axion field and the subscript 0 stands for its vacuum expectation value; T_s is the fundamental string tension in string metric (with l_s the string length scale)

$$T_s = \frac{1}{2\pi\alpha'} = \frac{1}{2\pi l_s^2}. \quad (3)$$

Using the level matching condition $N_R - N_L = nl$ and the BPS condition $N_R = 0$ or $N_L = 0$, one finds the spectrum of IIB BPS states

$$M_B = \frac{n}{R_B} + 2\pi R_B T_{(q_1, q_2)} l. \quad (4)$$

The spectrum of the M theory compactified on a torus of modular parameter $\tau = \tau_1 + i\tau_2$ with radii R_1 and R_{11} ($\tau_2 = R_1/R_{11}$) is [22, 1]

$$M_M^2 = \left(\frac{N}{R_{11}}\right)^2 + \left(\frac{m - \tau_1 N}{R_1}\right)^2 + (A_M T_2^M n)^2 + \dots, \quad (5)$$

where $A_M = (2\pi R_{11})(2\pi R_1)$ is the area of the torus. The first two terms come from the Kaluza-Klein modes, the third from the winding modes and the contribution of membrane excitations are not written down because the quantum theory of the membrane is what we are going after. Comparing the spectra (1) and (5), one finds that the Kaluza-Klein (winding) modes in type IIB theory match with the winding (Kaluza-Klein) modes in M theory, if we make the identification $m = q_1 l$, $N = q_2 l$, and ¹

$$R_B = 1/(A_M T_2^M), \quad (6)$$

$$R_B = l_s^2/R_1, \quad (7)$$

$$\chi_0 = \tau_1, \quad (8)$$

$$g_B = R_{11}/R_1 (= 1/\tau_2). \quad (9)$$

Therefore the modular parameter of the torus is identified with the vacuum expectation value of the complex field $\chi + ie^{-\phi}$, where ϕ is the dilaton field and $g_B = e^{\phi_0}$.

Here the consistency of eqs. (6) and (7) requires the relation

$$T_s = 2\pi R_{11} T_2^M, \quad (10)$$

implying that a fundamental IIB string is identified with a membrane wrapped on R_{11} . Therefore, given the parameters of M theory, the parameters of type IIB theory are determined by eqs. (6), (8-10). Conversely, given the parameters of type IIB theory, the parameters of M theory are determined by:

$$R_1 = \frac{l_s^2}{R_B}, \quad R_{11} = g_B \frac{l_s^2}{R_B}, \quad T_2^M = g_B^{-1} R_B T_s^2. \quad (11)$$

This duality also matches a membrane wrapped around a cycle of the torus in M theory with a string in type IIB theory: A cycle on the torus is specified by two mutually prime integers (q_1, q_2) with a minimal length

$$L_{(q_1, q_2)} = 2\pi R_{11} \left((q_1 - \tau_1 q_2)^2 + \tau_2^2 q_2^2 \right)^{1/2}. \quad (12)$$

¹Throughout this paper all quantities appearing in the same equation are given with respect to the same metric.

Hence the tension of a IIB string of charge (q_1, q_2) is $T_{(q_1, q_2)} = L_{(q_1, q_2)} T_2^M$, in agreement with the above relations.

Once the IIB /M duality is justified, the spectrum for M theory can be completed by looking at the IIB spectrum (4) [1]:

$$M_M = \left(\left(\frac{N}{R_{11}} \right)^2 + \left(\frac{m - \tau_1 N}{R_1} \right)^2 \right)^{1/2} + A_M T_2^M n, \quad (13)$$

for arbitrary integers N , m and n . Comparing this with eq. (5), we see that the second term includes both the contributions from winding modes and excitations on the membrane. To compare this spectrum to that of M(atric) theory, we need to go to the IMF by boosting in the direction of R_{11} so that $P_{11} \equiv N/R_{11} \gg m/R_1$. (This Lorentz transformation corresponds to a change in the RR-charge q_2 in type IIB theory to a large number, which can be achieved by an $SL(2, \mathbf{Z})$ symmetry transformation.) Thus, in the IMF, the spectrum in M theory appears to be

$$M_M = \frac{N}{R'_{11}} + \frac{R'_{11}}{2N} \left(\frac{m}{R'_1} \right)^2 - \frac{\tau_1 m}{\tau_2 R'_1} + A_M T_2^M n + \dots, \quad (14)$$

where (see Fig.1)

$$R'_{11} = \frac{\tau_2}{|\tau|} R_{11}, \quad R'_1 = \frac{|\tau|}{\tau_2} R_1. \quad (15)$$

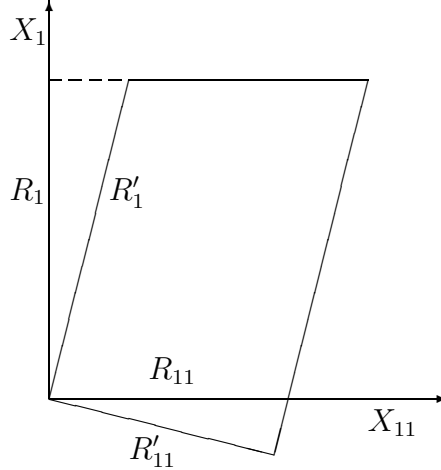


Figure 1: Longitudinal Torus

We note that the fourth term in eq. (14) is finite in the limit $N \rightarrow \infty$, without a factor of $1/P_{11}$ in front of it, showing that the energy of excitations on a longitudinal membrane scales as P_{11} under a boost in the longitudinal direction [14].

In next section we will try to identify the spectrum (14) in the M(atr)ix theory compactified on a circle.

3 M(atr)ix Theory and Compactification

In this section, we first review the uncompactified M(atr)ix theory, with an eye on the guiding role of IIA /M duality and on its IMF nature. Then, we will describe compactification of the theory on a circle, *not necessarily perpendicular* to the longitudinal direction that defines the IMF.

3.1 IIA /M Duality and Uncompactified M(atr)ix Theory

The BFSS action for M theory in the IMF is a large N limit of the supersymmetric matrix quantum mechanics obtained by dimensionally reducing the supersymmetric $U(N)$ Yang-Mills action from $9 + 1$ dimensions to $0 + 1$ dimension [7]:

$$S = \frac{1}{g} \int dt \operatorname{tr} \left(-\frac{1}{2} \nabla_0 A_i \nabla_0 A_i + \frac{1}{4} [A_i, A_j] [A_i, A_j] + \frac{i}{2} \bar{\Psi} (\gamma^0 \nabla_0 \Psi + \gamma^i [A_i, \Psi]) \right), \quad (16)$$

where $i, j = 1, 2, \dots, 9$ and $\nabla_0 = \partial_t + [A_0, \cdot]$. A_μ and Ψ are anti-hermitian $N \times N$ matrices; A_μ^a are real and Ψ^a are Majorana-Weyl spinors in 10 dimensions.

This matrix action was originally suggested [24] for a regularized supermembrane in eleven dimensions, since the $U(N)$ gauge symmetry

$$A_0 \rightarrow U A_0 U^\dagger + U \partial_t U^\dagger, \quad A_i \rightarrow U A_i U^\dagger, \quad \Psi \rightarrow U \Psi U^\dagger, \quad (17)$$

can be interpreted as the area-preserving diffeomorphism group of a membrane in the large N limit.

In the context of string theory, this action describes dynamics of N D0-branes in type IIA theory [25]. In the temporal gauge $A_0 = 0$, using the hermitian matrices $X_i = -i A_i$ ($i = 1, 2, \dots, 9$), the Hamiltonian is

$$H = \operatorname{tr} \left(\frac{g}{2} \Pi_i^2 - \frac{1}{4g} [X_i, X_j]^2 + \frac{1}{2g} \bar{\Psi} \gamma^i [X_i, \Psi] \right). \quad (18)$$

A minimum of the potential term in the Hamiltonian is reached when $\Psi = 0$ and all the X_i 's can be simultaneously diagonalized. Introducing

$$X_i = T_s x_i, \quad (19)$$

then $(x_i)_{\alpha\alpha}$ ($\alpha = 1, 2, \dots, N$) is interpreted as the i -th coordinate of the α -th D0-brane. An off-diagonal entry $(x_i)_{\alpha\beta}$ ($\alpha \neq \beta$) represents the effects due to open strings stretched between the α -th and the β -th D0-brane. Hence the energy of a stretched string, given by the string tension times the distance between the D0-branes, should equal the mass of the field $(x_i)_{\alpha\beta}$ in the action [25]. The coefficient of the action (16) is thus $1/g = T_0/T_s^2$, where T_0 is the D0-brane tension (the D-particle mass).

In accordance with the well-known IIA/M duality between the M theory compactified on S^1 (with radius R_{11}) and type IIA theory [26, 27, 22], membranes wrapped around S^1 are identified with fundamental IIA strings, and unwrapped membranes with D2-branes. The string tension and D2-brane tension are therefore related to the membrane tension T_2^M by $T_s = 2\pi R_{11} T_2^M$ and $T_2 = T_2^M$. Recall that the Dp-brane tension in type II string theory is given by [28, 29]

$$T_p = 1/((2\pi)^p g_s l_s^{(p+1)}), \quad (20)$$

where g_s is the string coupling g_A (g_B) in type IIA (IIB) theory for p even (odd). Therefore the compactification radius R_{11} and the membrane tension T_2^M can be given in terms of the IIA parameters (g_A, l_s) as

$$R_{11} = g_A l_s, \quad T_2^M = \frac{1}{(2\pi)^2 g_A l_s^3}, \quad (21)$$

or conversely,

$$g_A = 2\pi R_{11}^{3/2} (T_2^M)^{1/2}, \quad l_s = \frac{1}{2\pi R_{11}^{1/2} (T_2^M)^{1/2}}. \quad (22)$$

(The Planck length l_p in M theory is defined by $T_2^M = 1/((2\pi)^2 l_p^3)$, implying that $l_p = g_A^{1/3} l_s$.) It follows that

$$\frac{T_s^2}{g} = T_0 = \frac{1}{R_{11}}. \quad (23)$$

According to eq. (18), the center-of-mass kinetic energy of N D0-branes is

$$\frac{R_{11}}{2N} \sum_i (p_i^{com})^2, \quad (24)$$

where p_i^{com} is the conjugate momentum of the center-of-mass position $x_i^{com} = tr(x_i)/N$. The prefactor has an interpretation in M theory as the momentum of N partons (D0-branes) in the compactified direction: $P_{11} = N/R_{11}$. So the Hamiltonian (18) is understood as the light-cone energy with P_{11} very large, implying the eleven dimensional Lorentz invariance with the IMF expansion of kinetic energy: $K = (P_{11}^2 + \sum P_i^2)^{1/2} = P_{11} + (\sum_i P_i^2)/2P_{11} + \dots$.

In the limit $R_{11} \rightarrow \infty$ with l_p kept fixed, type IIA theory goes to the strong coupling limit while its dual theory – M theory – becomes uncompactified in eleven dimensions and are dominated by massless D0-branes (supergravitons). *Guided by the IIA /M duality*, the BFSS formulation of M(atric) theory [7] just postulates that in the limit both R_{11} and N/R_{11} going to infinity, the Hamiltonian (18) describes the uncompactified M theory in the IMF.

Here R_{11} is used as an infrared cutoff for the uncompactified theory: All winding modes are supposed to be thrown away, except those which wind R_{11} at most once corresponding to longitudinal branes.

To test the IIB /M duality we revisited in last section, normally one would consider compactification of M(atric) theory on a transverse torus [7, 30, 9, 10, 19], which needs to examine a $(2 + 1)$ -dimensional quantum gauge theory. However, as we are going to demonstrate, it is more instructive to test IIB /M duality in the M(atric) theory compactified only on a *circle* which, by combining the usual IIA /IIB and IIA /M dualities, is expected to be the strong coupling limit of type IIB string theory. One expects to gain interesting insights into physics in the IMF formulation from this study, because it will involve longitudinal membranes that wrap around R_{11} .

3.2 M(atric) Theory Compactified on an Oblique Circle

Now let us consider the M(atric) theory compactified on a circle, which is normally [7, 30, 10] taken to be in a direction, say X_1 , perpendicular to the longitudinal X_{11} , with radius R_1 . For our purpose, it is necessary to incorporate moduli parameters for equivalent IIB string theory, which needs to consider the compactification on an *oblique* circle, in a tilted X'_1 -direction in the $X_1 - X_{11}$ plane, with radius R'_1 . This makes sense in M(atric) theory. From the target-space point of view, at least in low energy supergravity, what is relevant is the Kaluza-Klein metric [22, 1] which gives rise to the moduli parameters of IIB strings. From the world-volume point of view, the resulting D1-brane action will be defined on the dual circle, whose radius is essential for quantization of the momentum modes of the D1-brane.

First recall the usual case with S^1 in the X_1 -direction. By gauging a discrete subgroup of $U(N)$ representing periodic translations in X_1 , the D0-brane action in the compactified space can be written as a D1-brane action on the dual circle [7, 30, 10]. In accordance with IIA /IIB T-duality, the winding modes around the circle for open strings stretched between D0-branes become the discretized momentum modes for D1-branes on the dual circle, while the compactified coordinate X^1 of the target space

turns into the covariant derivative with a gauge field A_1 on the worldsheet, leading to the action (in the temporal gauge) [30]:

$$S = \int dt \int_0^{2\pi R_B} \frac{dx}{2\pi R_B} \frac{1}{2g} \text{tr} \left(\dot{X}_i^2 - \dot{A}_1^2 - (\nabla_1 X_i)^2 + \frac{1}{2} [X_i, X_j]^2 + i\bar{\Psi}(\gamma^0 \dot{\Psi} + \gamma^1 \nabla_1 \Psi + \gamma^i [iX_i, \Psi]) \right), \quad (25)$$

where $i = 2, 3, \dots, 9$, $\nabla_1 = \partial_x + [A_1, \cdot]$ and $R_B = l_s^2/R_1$ is the dual radius of R_1 , exactly as required by IIA/IIB duality.

Now let us consider the case of an oblique S^1 in a tilted X'_1 -direction. In low-energy supergravity, the Kaluza-Klein metric in such coordinates will contain a gauge field in the X'_1 -direction. This gauge field can be gauged away except its Wilson line (or holonomy) degree of freedom. Motivated by this observation, in the dual description, we expect the appearance of a constant background electric field E for the gauge field on the D1-brane world-sheet so that the field strength F_{01} is shifted to $(F'_{01} - E)$, which is $(\dot{A}'_1 - E)$ in the temporal gauge. Now we denote the gauge field as A'_1 , because it corresponds to the tilted X'_1 -coordinate and has a period R'_1 . Indeed we can see how this modification comes about by considering compactification on a slanted torus. This leads to (see Appendix) the following modified action:

$$S = \int dt \int_0^{2\pi R_B} \frac{dx}{2\pi R_B} \frac{1}{2g'} \text{tr} \left(\dot{X}_i^2 - (\dot{A}'_1 - E)^2 - (\nabla_1 X_i)^2 + \frac{1}{2} [X_i, X_j]^2 + i\bar{\Psi}(\gamma^0 \dot{\Psi} + \gamma^1 \nabla_1 \Psi + \gamma^i [iX_i, \Psi]) \right), \quad (26)$$

where $\nabla_1 = \partial_x + [A'_1, \cdot]$; $R_B = l_s^2/R_1$ is unchanged, while

$$E = -i\lambda T_s \equiv -i\frac{\tau_1}{\tau_2} T_s, \quad \frac{T_s^2}{g'} = \frac{1}{R_{11}^2}. \quad (27)$$

Here $\tau = \tau_1 + i\tau_2$ is the modular parameter of the slanted longitudinal torus with radii R_1 and R_{11} ; the relations between (R'_1, R'_{11}) and (R_1, R_{11}) are exactly those of eq. (15). Note that the coefficient g' is defined as (23) with R_{11} replaced by R'_{11} , while the change of (R_1, R_{11}) to (R'_1, R'_{11}) does not affect the area A_M . Moreover, here the values of the field A'_1 are ranged between 0 and $2\pi R'_1 T_s^2$, with R'_1 just the radius of the $(0, 1)$ cycle on the slanted torus (see eq. (12)). Using eqs. (7) and (19), one finds that if eq. (9) is used to define g_B , then the prefactor of the action (26) is precisely the tension $T_{(0,1)} = |\tau| T_s$, appropriate for a IIB string of charge $(0, 1)$.

Comparing this to the action (25), we see that the term \dot{A}_1^2 has been replaced by $(\dot{A}'_1 - E)^2$, with the background gauge field E given by the above relation. In addition

to a constant term, this modification leads to adding a topological term of the form $-i\lambda \int \dot{A}'_1$ to the D1-brane action. This is an analogue of the θ -term in two dimensional gauge theory [31].

By imposing periodic boundary conditions in time, a change in λ by b results in a change in the action by $-i2\pi|\tau|^2 b/\tau_2$ times an integer. So the period for λ in the path integral measure e^{iS} is $\tau_2/|\tau|^2$.

We note that this moduli-dependent, topological term recently also appears in ref. [32] for the action of a D1-brane. It was used there to recover the fundamental string action by electric-magnetic duality. Here in our treatment this term is just right, see next section, to reproduce correctly the moduli-dependent term, the third term in eq. (14), in the IMF spectrum of M theory. We also note that the action (26) has an additional constant term proportional to E^2 , which is just right for the Hamiltonian to have a minimum of zero energy, in consistency with unbroken supersymmetry.

4 IIB /M Duality in M(atrix) Theory

We propose to interpret the M(atrix) theory action (26) as describing a system of N D-strings in (strongly coupled) type IIB theory compactified on a circle.

4.1 Spectrum of IIB BPS States

As evidence for this equivalence, we now show that the spectrum (14) expected from IIB /M duality can be found in the M(atrix) theory.

First, the kinetic energy of the $U(1)$ factor of A'_1 , $tr(A'_1)/N$, gives the second term in (14), which is simply part of the matching (24) mentioned in last section. In addition, the contribution of the topological term (due to the Wilson line) to the Hamiltonian just matches the third term in (14) as well, because of the relation (27) between E and τ_1/τ_2 .

Now consider the configurations which satisfy $[X_i, X_j] = 0$ and $\Psi = 0$ where one can simultaneously diagonalize the X_i 's. The action (26) becomes proportional to the free action $\sum_{\alpha} ((\dot{X}_i)_{\alpha\alpha}^2 - (\partial_x X_i)_{\alpha\alpha}^2)$. By Fourier expansion: $X_i = \sum_k X_{ik} e^{ikx/R_B}$, one finds that the excitation (or oscillation) modes of X_i on the closed D1-brane (D-string) have the spectrum $M = n/R_B = 2\pi T_s R_1 n$ upon quantization. The operator n is defined by $n = \sum_{ik\alpha} k N_{ik\alpha}$, where $N_{ik\alpha}$ is the number operator for the mode $(X_{ik})_{\alpha\alpha}$. When $[X_i, X_j]^2$ and other terms are included, there are interactions between D-strings. However, we expect a non-renormalization theorem for the spectrum of these states,

because they correspond to BPS states of IIB strings. Obviously this part of energy should be identified with the fourth term in (14), or equivalently with the first term in (4).

We observe that in the present IMF formulation, this part of the energy, which involves the excitations of the D-string, does *not* have a prefactor of $1/(2P_{11})$, in contrast to the usual IMF energies for purely transverse excitations. This implies that the proper interpretation of this part of energy in M theory should be attributed to excitations on a (longitudinal) membrane wrapped on the eleventh direction, just as expected from IIB/M duality. Note that in this argument we have taken advantage of the IMF formulation, without the need of constructing a semi-classical longitudinal membrane.

One can check that the topological configuration of A'_1 and the oscillation modes of X_i are both indeed BPS states in M(atrrix) theory.² The eleven dimensional supersymmetry transformations in the IMF consist of the dynamical part

$$\delta A_\mu = \frac{i}{2} \bar{\epsilon} \gamma_\mu \Psi, \quad \delta \Psi = -\frac{1}{4} F_{\mu\nu} \gamma^{\mu\nu} \epsilon, \quad (28)$$

where $F^{0i} = \dot{X}^i$, $F^{1i} = \nabla_1 X^i$ and $F^{ij} = [X^i, X^j]$, and the kinematical part

$$\delta A_\mu = 0, \quad \delta \Psi = i\tilde{\epsilon}, \quad (29)$$

where ϵ and $\tilde{\epsilon}$ are both Majorana-Weyl spinors times a unit matrix. Each part of the supersymmetry has 16 generators and the total supersymmetry has 32 generators. It turns out that the supersymmetry algebra involves central terms which can be interpreted as various RR charges [14]. The topological configuration of A'_1 preserves one half of the total SUSY as a linear combination of the dynamical part and the kinematical part. The associated charge is simply the Kaluza-Klein momentum P_1 proportional to m . The purely left-moving (or right-moving) oscillation modes preserve one quarter of the total SUSY (half of the dynamical part) and give one nonzero RR charge Z^1 (defined in Ref.[14] as a central term in the SUSY algebra) proportional to k , corresponding to branes wrapped around R_{11} .

It is instructive to compare the above identification of the spectrum for a longitudinal membrane with that for a transverse membrane [7]. The configuration in M(atrrix) theory that represents a membrane wrapped n times around a transverse torus with radii R_1 and R_2 is given by $x_1 = R_1 p$, $x_2 = R_2 q$, with p and q satisfying $[p, q] = 2\pi i n/N$. (While the representation of the canonical commutation relation can only be realized in infinite dimensional representations, the commutation relation above makes sense with

² We thank M. Li for discussions on this matter.

the understanding that x_i ($i = 1, 2$) lives on a circle of radius R_i and that the right hand side is appropriately normalized by the dimension of the representation N .³⁾ As easy to verify, it is the potential term $\frac{1}{4}[x_i, x_j]^2$ in the light-cone Hamiltonian (18) that correctly reproduces the membrane spectrum $A_M T_2^M n$, where $A_M = (2\pi R_1)(2\pi R_2)$ is the area of the torus, for this configuration.

In this discussion, to match the IIB spectrum, it is necessary to have the membrane wound around the R_{11} only once. This is in agreement with the use of R_{11} in M(atr)ix theory as a cutoff, so that longitudinal branes are allowed to wind it only once! In addition, this is exactly what has been conjectured by Schwarz in his discussions on IIB /M duality [1], namely that in M theory the membrane wrapped on a torus should select a preferred cycle in which it is wrapped many times, and this preferred direction must be the one defined by the IIB theory Kaluza-Klein excitations, which is nothing but the R_1 direction! This can be argued as follows. The IIB Kaluza-Klein modes by T-duality are IIA string winding modes around R_1 . The IIA strings are membranes wound around R_{11} only once by IIA /M duality. Hence the IIB Kaluza-Klein modes are membranes wound around R_{11} once and R_1 an arbitrary number of times.

4.2 Relations among IIA /IIB /M Parameters

The above spectrum matching can also be understood from IIA /IIB and IIA /M dualities. Let us recall that the quantity $(X_{ik})_{\alpha\alpha}$ has a dual interpretation in IIA or IIB language. In IIA language, when compactifying D0-branes on a circle of radius R_1 , an open string wound k times around the circle with both ends on the same D0-brane labelled α has the energy $2\pi k T_s R_1$. Such a winding mode of an open string is known to be represented by $(X_{ik})_{\alpha\alpha}$ [30]. On the other hand, in the dual (IIB) language, $(X_{ik})_{\alpha\alpha}$ represents an oscillation mode on the D-string, with energy k/R_B which is identical to $2\pi k T_s R_1$. On the other hand, because strings in type IIA theory are thought of, by IIA /M duality, as membranes wound around R_{11} , so we are again led to the identification in the last subsection of the D-string oscillation modes with excitations on the longitudinal membrane.

Thus, what we have here is IIA /IIB /M triality, i.e. the M(atr)ix theory compactified on a circle is equivalent to either the strong coupling limit of type IIA or type IIB theory, each compactified on a circle too. The IIA /M duality and IIB /M duality we considered separately in the above are linked by the usual IIA /IIB duality.

If we consider the IIA /M duality for the M theory compactified on a torus and

³ We thank B. Zumino for pointing this out to us.

IIA on a circle, the IIA /M relations for this case are simply (21-22) together with

$$R_A = R_1, \quad (30)$$

where R_A is the radius of the circle in IIA theory. The IIA /IIB duality identifies D p -branes in IIA with D($p \pm 1$)-branes in IIB theory by wrapping or unwrapping. Therefore we have (7) and

$$g_B = g_A \frac{l_s}{R_A}. \quad (31)$$

It can be easily checked that these relations, required by dualities of each pair of IIA, IIB and M theories, are satisfied in the M(atrrix) theory compactified on a circle. Also using any two of the dualities, one can derive uniquely the third one. If one uses the dualities to go cyclicly from one theory through the other two to return to the original theory, one finds that the parameters are unchanged after the journey. If it were not the case it would mean that there exist new self-dualities in these theories.

Because R_{11} is to be treated as a cutoff, it should be much larger than any other length scale in the theory. Hence in the limit $R_{11} \rightarrow \infty$, the matrix model gives the S^1 -compactified M theory dual to the type IIB theory compactified on the dual circle in the strong coupling limit according to (9).

4.3 $SL(2, \mathbf{Z})$ Duality of IIB Theory

The $SL(2, \mathbf{Z})$ duality of IIB theory transforms a (q_1, q_2) string to a string with charge (q'_1, q'_2) given by

$$\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} \rightarrow \begin{pmatrix} q'_1 \\ q'_2 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}, \quad (32)$$

where $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is an $SL(2, \mathbf{Z})$ matrix. The coupling and string tension transform as

$$g_B \rightarrow g'_B = |c\tau + d|^2 g_B, \quad (33)$$

$$T_s \rightarrow T'_s = |c\tau + d| T_s. \quad (34)$$

These agree, by the IIB /M duality relations (6-11), with the modular transformation

$$\tau \rightarrow \tau' = \frac{a\tau + b}{c\tau + d} \quad (35)$$

of the torus in M theory, on which the theory is compactified. For the modular transformation to be a geometric symmetry of M theory, the area A_M of the torus is to be

fixed and thus the radii transform as

$$R_1 \rightarrow \frac{R_1}{|c\tau + d|}, \quad R_{11} \rightarrow |c\tau + d|R_{11}. \quad (36)$$

The spectrum in M theory, $((N/R_{11})^2 + ((m - \tau_1 N)/R_1)^2)^{1/2}$ is invariant under the transformation (35), (36) and

$$\begin{pmatrix} m \\ N \end{pmatrix} \rightarrow \begin{pmatrix} m' \\ N' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} m \\ N \end{pmatrix}. \quad (37)$$

In the IMF description of the M(atrrix) theory compactified on a circle, we do not expect to have the full $SL(2, Z)$ symmetry, since the longitudinal direction is preferred, and the limit of R_{11} may be different from that of R_1 . However, the theory may be invariant under a subgroup. We have checked that if we make the transformation (35-37), then the invariance of the IMF spectrum (14) requires that $a = 1, b = 0$ and $d = 1$. So the remnant symmetry is

$$\begin{pmatrix} 1 & 0 \\ c & 1 \end{pmatrix} \quad (38)$$

where c is an integer.

4.4 Finiteness of Spectrum and Limits of Parameters

For the spectrum M_M , eq. (14), we obtained in Sec.4.1 for M(atrrix) theory to make sense in the limit $R_{11} \rightarrow \infty$, other parameters in the theory have to take appropriate limits accordingly. To be more precise about these limits, one should consider only dimensionless quantities. For instance,

$$r_B = R_B/l_s, \quad r_B^c = g_B^{-1/4} R_B/l_s \quad (39)$$

are the IIB radius measured in the IIB string metric and the canonical metric, where $l_s = 1$ and $g_B^{1/4} l_s = 1$, respectively;

$$r_1 = R_1/l_p, \quad r_{11} = R_{11}/l_p \quad (40)$$

are the values of R_1 and R_{11} measured in the eleven-dimensional Planck units. Similarly, the finiteness of the spectrum is to be considered with respect to a certain system of units.

In the M theory it is natural to measure everything in terms of the Planck scale l_P , so the dimensionless spectrum is $m_P = M_M l_P$. In IIB theory one can choose to use the

string metric or the canonical metric, where the spectrum appears to be $m_s = M_M l_s$ and $m_s^c = g_B^{1/4} M_M l_s$, respectively. The results in the IIB canonical metric are the same as in the Planck units. In the following we discuss separately in the Planck units and string units the appropriate limits of various parameters in the theory for the spectrum to be finite.

In the Planck units, for both the third and the fourth terms in eq. (14) to have finite limits, we need

$$r_B^c \sim \text{finite}, \quad r_1 \sim r_{11}^{-1}, \quad \tau_1 \sim r_{11}^{-3}, \quad \tau_2 \sim r_{11}^{-2}. \quad (41)$$

As a consequence, the parameter $\lambda \equiv \tau_1/\tau_2$ does not have a finite period since its period $\tau_2/|\tau|^2$ goes to infinity. It is also easy to check that the modular parameter τ_1/τ_2 of our longitudinal torus, scaled by $(l_p/l_s)^2$ so that it has a finite limit, is invariant under the remnant subgroup of $SL(2, \mathbf{Z})$ mentioned in the previous subsection. In this case one has $r_1 \rightarrow 0$. This gives a version of the M theory dual to the IIB theory compactified on a finite circle in the strong coupling limit. The part of the spectrum considered above has a finite limit in both the Planck units and the IIB canonical metric. The opposite extremum $r_1 \rightarrow \infty$ is just the original case considered in Ref.[7].

In the string metric we need

$$r_B \sim \text{finite}, \quad \tau_2 \sim r_{11}^{-3/2} \quad (42)$$

for arbitrary τ_1 . In this case it is possible to choose $\tau_1 \sim \tau_2^{1/2}$ so that the parameter λ has a finite period. The modular parameter τ_1/τ_2 can have arbitrary limit by choosing the limit of τ_1 . But in any case τ_1/τ_2 scaled by an appropriate power of l_p/l_s is invariant under the remnant subgroup. Here we get a strong coupling limit of IIB theory compactified on a circle with a finite radius in string units but zero radius in the canonical metric.

It is certainly possible to take other limits. For instance, one can take the units such that R_1 is finite, then one needs $T_2^M \rightarrow 0$. This is a special limit in the IIB /M duality which is the strong coupling limit of IIB theory (or by T-duality, IIA theory) compactified on a circle as well as the weak tension limit in the M theory compactified on the dual circle.

5 Affine Lie Group Symmetry

An accidental affine Lie algebraic structure appears in M(atric) theory when it is compactified on a circle. In Ref.[33], WZW models of Lie group valued fields on two

dimensional spacetime are rewritten as WZW models of affine Lie group valued fields on one dimensional world history. In the same spirit, by imposing periodic boundary conditions,⁴ we decompose the x dependence of fields into their Fourier modes:

$$A_\mu = \alpha_{\mu m}^a T^a e^{imx/R}, \quad \Psi = \psi_m^a T^a e^{imx/R}, \quad (43)$$

where $R = R_B$. Since A_μ and T^a are both antihermitian, we have $\alpha_{\mu m}^{a*} = \alpha_{\mu-m}^a$ and $\psi_m^{a*} = \psi_{-m}^a$ under complex conjugation denoted by $*$ in the Majorana representation.

In the action (25) only the traces $\kappa^{ab} = \text{tr}(T^a T^b)$ of quadratic products of Lie algebra generators are used, which are simply the Killing metric up to normalization. This is the key property that leads to the affine Lie group symmetry.

Let $T_m^a = T^a e^{imx/R}$. They satisfy the loop algebra

$$[T_m^a, T_n^b] = f^{abc} T_{m+n}^c. \quad (44)$$

The trace $\underline{\text{tr}}$ on the loop algebra

$$\underline{\text{tr}}(T_m^a T_n^b) = \kappa^{ab} \delta_{m+n}^0 \quad (45)$$

is equivalent to

$$\underline{\text{tr}}(\cdot) = \int_0^{2\pi R} \frac{dx}{2\pi R} \text{tr}(\cdot). \quad (46)$$

By adding the generator D to the loop algebra according to

$$[D, T_m^a] = m T_m^a, \quad (47)$$

one can rewrite the action (25) in terms of loop algebra valued quantities

$$A_\mu = \alpha_{\mu m}^a T_m^a, \quad \Psi = \psi_m^a T_m^a. \quad (48)$$

It is

$$\begin{aligned} S = \int dt \frac{1}{2g} \underline{\text{tr}} \Big(& -\dot{A}_i^2 - \dot{A}_1^2 + \left[\frac{iD}{R} + A_1, A_i \right]^2 + \frac{1}{2} [A_i, A_j]^2 \\ & + i\bar{\Psi}(\gamma^0 \dot{\Psi} + \gamma^1 \left[\frac{iD}{R} + A_1, \Psi \right] + \gamma^i [A_i, \Psi]) \Big), \end{aligned} \quad (49)$$

where $i = 2, 3, \dots, 9$.

As a result, the $U(N)$ gauge symmetry becomes a loop group symmetry. This loop group symmetry is no longer a local symmetry in the usual sense since there is

⁴One can also impose twisted boundary conditions which would lead to twisted affine Lie groups.

no coordinate dependence anymore. It is still a gauge symmetry in the sense that all observables are required to be loop group invariants.

It is interesting that this action can be written as an action for affine Lie algebra valued quantities. Define \hat{A}_i and $\hat{\Psi}$ to be affine Lie algebra valued quantities. Let

$$\hat{A}_i = iz_i K + \alpha_{im}^a T_m^a, \quad \hat{\Psi} = i\eta K + \psi_m^a T_m^a, \quad (50)$$

where K and T_m^a are affine Lie algebra generators satisfying

$$[T_m^a, T_n^b] = f^{abc} T_{m+n}^c + m\kappa^{ab} \delta_{m+n}^0 K, \quad (51)$$

$$[T_m^a, K] = 0. \quad (52)$$

The hermitian conjugation on this algebra can be defined by

$$K^* = K, \quad T_m^{a*} = -T_{-m}^a. \quad (53)$$

For a more detailed discussion of affine Lie algebras see for instance Refs.[34][35][36].

By adding the generator D one obtains the extended affine Lie algebra, sometimes simply referred to as the affine Lie algebra [35, 36] satisfying (47) and

$$D^* = D, \quad [D, K] = 0. \quad (54)$$

The Killing metric of the affine Lie algebra is determined algebraically up to normalization except $Tr(DD)$. But since the generator D is defined by the commutation relations and hermiticity only up to a shift by K : $D \rightarrow D + cK$ for a real number c , we can always make such a shift so that the result is the following:

$$Tr(T_m^a T_n^b) = \kappa^{ab} \delta_{m+n}^0, \quad (55)$$

$$Tr(DK) = Tr(KD) = 1, \quad (56)$$

$$Tr(DD) = Tr(KK) = 0, \quad (57)$$

$$Tr(T_m^a D) = Tr(T_m^a K) = 0. \quad (58)$$

One can easily check that the action (49) remains the same if we replace \underline{tr} by Tr , A_μ and Ψ by \hat{A}_μ and $\hat{\Psi}$ everywhere in the action. For configurations with $\alpha_{\mu m}^a = \psi_m^a = 0$ for all $m \neq 0$ the action reduces to the effective action of D0-branes.

The action of affine Lie algebra valued quantities is invariant under K -shifts:

$$\hat{A}_i \rightarrow \hat{A}_i + a_i K, \quad \hat{\Psi} \rightarrow \hat{\Psi} + \lambda K, \quad (59)$$

hence we can “gauge” this symmetry so that z_i and η are not physical observables. Most importantly, the action is invariant under the affine Lie group transformation,

i.e., conjugation of \hat{A}_i and $\hat{\Psi}$ by unitary elements in the affine Lie group. Let $y_1 = 1$, $y_i = 0$ for $i \neq 1$, the infinitesimal version of this transformation is:

$$\delta_\epsilon \hat{A}_i = [\epsilon, iy_i D/R + \hat{A}_i], \quad \delta_\epsilon \hat{\Psi} = [\epsilon, \hat{\Psi}] \quad (60)$$

for a hermitian element in the affine Lie algebra $\epsilon = aD + bK + c_m^a T_m^a$. Explicitly, the coefficients in \hat{A}_i and $\hat{\Psi}$ transform as

$$\delta_\epsilon z_i = mc_m^a \kappa^{ab} \alpha_{i-m}^b, \quad (61)$$

$$\delta_\epsilon \eta = mc_m^a \kappa^{ab} \theta_{-m}^b, \quad (62)$$

$$\delta_\epsilon \alpha_{im}^a = ma \alpha_{im}^a - im c_m^a y_i / R + f^{abc} c_n^b \alpha_{i(m-n)}^c, \quad (63)$$

$$\delta_\epsilon \psi_m^a = ma \psi_m^a + f^{abc} c_n^b \psi_{(m-n)}^c. \quad (64)$$

This is equivalent to a loop group transformation together with a translation in x .

A highest weight representation of level k for the affine Lie algebra is given by a vacuum state satisfying $T_m^a |0\rangle = 0$ for $m > 0$ and $K|0\rangle = k|0\rangle$, with all other states in this representation obtained by having products of generators acting on the vacuum state. In a highest weight representation, one can always realize not only the affine Lie algebra but actually the semi-direct product of the affine Lie-algebra and the Virasoro algebra, which is given by (51-53) together with

$$[L_m, T_n^a] = -n T_{m+n}^a, \quad [L_m, K] = 0, \quad (65)$$

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12} m(m^2 - 1) \delta_{m+n}^0. \quad (66)$$

It is called Sugawara's construction, in which the generators of the Virasoro algebra L_m are realized by

$$L_m = \frac{1}{2k+Q} \kappa_{ab} \left(\sum_{n \leq 0} T_{m+n}^a T_{-n}^b + \sum_{n > 0} T_{-n}^b T_{m+n}^a \right), \quad (67)$$

where Q is the quadratic Casimir in the adjoint representation: $Q\delta^{ab} = f^{acd} f^{bcd}$, and the central element is $c = \frac{2kd}{2k+Q}$, where d is the dimension of the Lie algebra. From the commutation relations (65) one sees that L_0 can be identified with D .

When $c = 0$, the Virasoro algebra is the algebra of infinitesimal diffeomorphisms on a circle. Indeed the transformation of the Virasoro algebra on A_μ and Ψ induced from its action on the loop algebra (65) can be realized by the diffeomorphism generators $e^{imx/R} \frac{\partial}{\partial x}$ acting on the D-strings. It is tempting to think of the Virasoro algebra implicit in highest weight representations as a signal of the implicit existence of conformal field theory (strings) in the matrix model.

6 Discussions: Insights into Longitudinal Membranes

Previously IIB /M duality refers to the equivalence between the M theory compactified on a torus and type IIB superstring theory compactified on a circle. The recently proposed nonperturbative formulation of M(atrrix) theory makes it possible to discuss the equivalence between the M(atrrix) theory compactified on a circle and (the strong coupling limit of) type IIB theory also compactified on a circle. In this paper we establish this IIB /M duality in the M(atrrix) theory context. Several pieces of evidence we provide are described in Sec. 4, and summarized in the abstract and introduction.

Here we would like to concentrate on the insights we have gained into the IMF description of M(atrrix) theory, since our study involves the behavior of a longitudinal membrane.

No one has succeeded in constructing a longitudinal membrane, in the way a transverse membrane is constructed [7], and no one doubts the existence of a longitudinal membrane in M(atrrix) theory. From our study we have seen indeed the longitudinal membranes are hiding in the theory. By the IIB /M duality established above, properties of a membrane that wraps the longitudinal direction once can be extracted from those of the D-string obtained from compactification on a circle. These properties are:

- The excitations of a longitudinal membrane are identified with the quantum oscillation modes on the D-string, by noting that the light-cone energy of these modes are finite. This is in sharp contrast to the energy of a purely transverse membrane, which comes from the commutator potential term.
- For a longitudinal membrane wrapped on the longitudinal torus with modular parameter τ , its light-cone energy has a term dependent on the ratio τ_1/τ_2 , and independent of P_{11} , as expected from general grounds (see eq. (14)). The mechanism in M(atrrix) theory responsible for this energy is similar to that in a θ vacuum on the D-string, since the latter contains a topological term with τ_1/τ_2 as coefficient, analogous to the θ vacuum parameter.
- The properties of a longitudinal membrane we extract through studying D1-branes are always such that they can be viewed as the limit of a membrane wrapped on a transverse torus with one of the two cycles going to infinity. This fact provides us one more evidence for eleven dimensional Lorentz invariance of M(atrrix) theory.

In summary, we conclude that the properties of a membrane which wraps once

in the longitudinal direction can be extracted from those of a D-string obtained by compactifying M(atric) theory on a circle.

Another interesting way to look at the model considered in this paper is to interchange the roles of R_1 and R_{11} . Namely, previously we have $R_A = R_1$ and $g_A = R_{11}/l_s$; but now we set $R'_A = R_{11}$ and $g'_A = R_1/l_s$. The ten dimensions of IIA theory are thus the 0, 2, 3, \dots , 9, 11-th dimensions. If $\tau_1 = 0$, the first two terms in the spectrum (14) are the large P_{11} expansion of the relativistic kinetic energy $\sqrt{P_{11}^2 + (mT_0)^2}$ where mT_0 is the mass of m D0-branes. The fourth term in (14) is the winding energy of n IIA string wound around R_{11} once. With $R_{11} \rightarrow \infty$ and $R_1 \rightarrow 0$, this model should be equivalent to uncompactified IIA theory in the weak coupling limit (or its dual IIB theory).

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Note added: When the writing of this paper is about to finish, a preprint of T. Banks and N. Seiberg, hep-th/9702187, appears which, among other things, has some overlap with part of what we address here.

A Modular Parameter and Compactification on Torus

The general idea of compactification of M(atric) theory on a compact manifold [7, 30, 10] is to consider the compact manifold as the quotient of a covering space over a discrete group. The matrix model on the compact space is then obtained by taking the quotient of the $U(N)$ matrices over the discrete group, which is to be embedded in $U(N)$ as a subgroup.

For toroidal compactifications [30, 10], one needs to choose unitary matrices $\{U_i, i = 1, \dots, d\}$, where d is the dimension of the torus, commuting with each other so that the discrete group generated by them is isomorphic to the fundamental group of the torus \mathbf{Z}^d . The action of U_i on the coordinates is

$$U_i X^\mu U_i^\dagger = X^\mu + e_i^\mu, \quad (68)$$

where e_i^μ is the basis of the lattice whose unit cell is the torus. Obviously U_i is the operator translating all fields a whole cycle along e_i .

The matrix theory is then restricted to be invariant under the action of the U_i 's. This is most easily realized by viewing $e_\mu^i X^\mu$ as the covariant derivative $\frac{\partial}{\partial x^i} + A_i$ in the

direction of e^i , dual to e_i , and the U_i as $e^{2\pi i x^i}$, where x^i is a coordinate on the dual torus and so we have interchanged the role of coordinates and momenta. Naturally the matrix model becomes a $d + 1$ dimensional gauge field theory.

For a slanted two-torus with modular parameter $\tau = \tau_1 + i\tau_2$ and radii $R_1, R_2 = \tau_2 R_1$, it is convenient to introduce slanted coordinates (X'_1, X'_2) by

$$\begin{pmatrix} X'_1 \\ X'_2 \end{pmatrix} = \begin{pmatrix} 1 & -\frac{\tau_1}{\tau_2} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \quad (69)$$

out of the orthonormal coordinate system (X_1, X_2) . The nice thing about the X'_i 's is that they live on circles of radii R_i . The discrete group generated by U_1, U_2 with $U_1 U_2 = U_2 U_1$ acts on them simply as $U_i X'_j U_i^\dagger = X'_j + 2\pi \delta_{ij} R_j$. From the action $\frac{1}{2}(\dot{X}_1^2 + \dot{X}_2^2)$ one defines conjugate variables and finds

$$\begin{pmatrix} P'_1 & P'_2 \end{pmatrix} = \begin{pmatrix} P_1 & P_2 \end{pmatrix} \begin{pmatrix} 1 & \frac{\tau_1}{\tau_2} \\ 0 & 1 \end{pmatrix}. \quad (70)$$

It follows that the kinetic energy is $\frac{1}{2}(P_1^2 + P_2^2) = \frac{1}{2}(P_1'^2 + (P_2' - \frac{\tau_1}{\tau_2} P_1')^2)$, giving the spectrum of $\left(\frac{m_1}{R_1}\right)^2 + \left(\frac{m_2 - \tau_1 m_1}{R_2}\right)^2$.

The case of a longitudinal membrane can be inferred from this result. To do so, note that R_1 and R_2 here correspond to R_{11} and R_1 , respectively, in this paper for the longitudinal torus. Therefore the first two terms in eq. (5) follow. Further, the action $\frac{1}{2}(\dot{X}_1^2 + \dot{X}_2^2)$ can be written as $\frac{1}{2}(\dot{Y}_1^2 + (\dot{Y}_2 + \tau_1 \dot{Y}_1 / \tau_2)^2)$, where $Y_1 = \tau_2 X'_1 / |\tau|$ and $Y_2 = |\tau| X'_2 / \tau_2$. Applying this result to the longitudinal case in the infinite momentum frame ($\dot{Y}_1 = 1$), we see that the prescription is to replace R_{11} by R'_{11} and \dot{A}_1 by $(\dot{A}'_1 + i(\tau_1 / \tau_2) T_s)$ where A'_1 (corresponding to $i T_s Y_2$) is valued in the range $i T_s [0, 2\pi R'_1]$. This gives what we found in Sec.4.1.

References

- [1] J. H. Schwarz: "Lectures on Superstring and M Theory Dualities", hep-th/9607201.
- [2] A. Sen: "Unification of String Dualities", hep-th/9609176.
- [3] J. Polchinski: "TASI Lectures on D-branes", hep-th/9611050.

- [4] M. J. Duff: “M-Theory (the Theory Formerly Known as Strings)”, hep-th/9608117.
- [5] M. R. Douglas: “Superstring Dualities, Dirichlet Branes and Small Scale Structure of Space”, hep-th/96100041.
- [6] P. K. Townsend: “Four Lectures on M Theory”, hep-th/9612121.
- [7] T. Banks, W. Fischler, S. H. Shenker, L. Susskind: “M Theory as a Matrix Model: A Conjecture”, hep-th/9610043.
- [8] M. Berkooz, M. R. Douglas: “Five-Branes in M(atrix) Theory”, hep-th/9610236.
- [9] L. Susskind: “T Duality in M(atrix) Theory and S Duality in Field Theory”, hep-th/9611164.
- [10] O. J. Ganor, S. Ramgoolam, W. Taylor IV: “Branes, Fluxes and Duality in M(atrix)-Theory”, hep-th/9611202.
- [11] O. Aharony, M. Berkooz: “Membrane Dynamics in M(atrix) Theory”, hep-th/9611215.
- [12] G. Lifschytz, S. D. Mathur: “Supersymmetry and Membrane Interactions in M(atrix) Theory”, hep-th/9612087.
- [13] M. R. Douglas: “Enhanced Gauge Symmetry in M(atrix) Theory”, hep-th/9612126.
- [14] T. Banks, N. Seiberg, S. Shenker: “Branes from Matrices”, hep-th/9612157.
- [15] G. Lifschytz, “Four-Brane and Six-Brane Interactions in M(atrix) Theory”, hep-th/9612223.
- [16] I. I. Kogan, G. W. Semenoff, R. J. Szabo: “D-brane Configurations and Nicolai Map in Supersymmetric Yang-Mills Theory”, hep-th/9612251.

- [17] N. Kim, S.-J. Rey: “M(atrix) Theory on an Orbifold and Twisted Membrane”, hep-th/9701139.
- [18] D. A. Lowe: “Bound State of Type I’ D-Particles and Enhanced Gauge Symmetry”, hep-th/9702006.
- [19] S. Sethi, L. Susskind: “Rotational Invariance in the M(atrix) Formulation of Type IIB Theory”, hep-th/9702101.
- [20] D. Berenstein, R. Corrado: “M(atrix) Theory in Various Dimensions”, hep-th/9702108.
- [21] M. Rozali: “Matrix Theory and U-duality in Seven Dimensions”, hep-th/9702136.
- [22] J. H. Schwarz: “The Power of M Theory”, hep-th/9510086.
- [23] J. H. Schwarz: “An $SL(2, \mathbf{Z})$ Multiplet of Type IIB Superstrings”, hep-th/9508143.
- [24] de Wit, J. Hoppe, H. Nicolai: “On the Quantum Mechanics of Supermembranes”, *Nucl. Phys.* **B305** [**FS23**], 545-581 (1988).
- [25] E. Witten: “Bound States of Strings and p-Branes”, *Nucl. Phys.* **B460**, 335-350 (1996).
- [26] P. K. Townsend: “The Eleven-Dimensional Supermembrane Revisited”, hep-th/9501068, *Phys. Lett.* **B350**, 184 (1995).
- [27] E. Witten: “String Theory Dynamics in Various Dimensions”, hep-th/9503124, *Nucl. Phys.* **B443**, 85 (1995).
- [28] J. Polchinski, S. Chaudhuri, C. V. Johnson: “Notes on D-Branes”, hep-th/9602052.
- [29] S. P. de Alwis: “A Note on Brane Tension and M-theory”, hep-th/9607011.
- [30] W. Taylor IV: “D-Brane Field Theory on Compact Spaces”, hep-th/9611042.

- [31] E. Witten: “Theta Vacua in Two-Dimensional Quantum Chromodynamics”, *Nuovo Cim.* **51 A**, 325 (1979).
- [32] M. Aganagic, J. Park, C. Popescu, J. H. Schwarz: “Dual D-Brane Actions”, hep-th/9702133.
- [33] K. Clubok, M. B. Halpern: “The WZW Model as a Dynamical System on Affine Lie Groups”, *Int. J. Mod. Phys.* **A11**, 2167 (1996).
- [34] P. Goddard, D. Olive: “Kac-Moody Algebra and Virasora Algebra in Relation to Quantum Physics”, *Int. J. Mod. Phys.* **A1**, 303-414 (1986).
- [35] Z.-X. Wan: “Introduction to Kac-Moody Algebra”, World-Scientific (1991).
- [36] J. Fuchs: “Affine Lie Algebra and Quantum Groups”, Cambridge Univ. Press (1992).